Mathematics 126 Final Exam Dec. 8, 2003

- Calculators are allowed *only* for numerical calculations.
- There are three sheets of scratch paper attached at the end of the exam. Use them and but do not tear them off the exam in doing so, and hand them in together with the exam.
 - Show your work; clearly write down each step in your calculations/reasonings.

- ${\bf 1.}$ Evaluate the following definite and indifinite integrals.
- a)

$$\int_{1}^{e} \frac{(\ln x)^2}{x} \ dx$$

b)

$$\int_2^3 \frac{1}{x^2 - 1} \ dx$$

c)
$$\int_0^1 \sqrt{5x+4} \ dx$$

$$\int x \ e^x \ dx$$

e)
$$\int_{2}^{6} \sqrt{x-2} \ dx$$

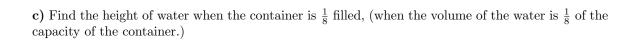
Note: this is an improper integral.

f)
$$\int_0^1 \frac{x^3}{x^2 + 1} \ dx$$

2.	There is an	upside	-down	pyrami	d-shape	ed contain	er who	se s	quare b	oase	has	its side-le	ength	20 i	n.
an	d its height	10 in	Let h	be the	height	measured	from t	the	bottom	of t	the o	container,	that	is th	ıe
tip	of the pyra	mid is	at $\{x =$	= 0}											

a) Write down the area A(x) of the cross-section of the container at height x.

b) Write down the volume V(h) of water of height h in the contianer as an integral of A(x), and evaluate the integral.



d) Find the height h_0 so that $V'(h_0)=64$. (Hint: apply the fundamental theorem of calculus to V(h).)

3. Sketch the region enclosed by the curves y = |x|, $y = x^2 - 2$. Find the area of the region.

- **4.** Let P = (1, 0, -1) and Q = (2, 1, 2).
- a) Take the dot product of the two vectors \vec{OP} and \vec{OQ} . What can you say about the angle between the two vectors?

b) Take the cross product of the two vectors \vec{OP} and \vec{OQ} .

c) Find an equation of the plane containing O, P and Q.

5. Determine whether each *series* converges or not. Write down your reasoning (what test you used etc..)

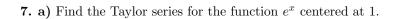
$$\sum_{n=1}^{\infty} a_n, \text{ where } a_1 = 1 \text{ and } a_{n+1} = \frac{2n^2}{3n^2 + n} a_n.$$

$$\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{1 + 2\sqrt{n}}$$

6.	a)	Using	the	integral	of the	function	1/x,	show	that	the	n-th	partial	sum	s_n	of	the	harmo	onic
ser	ies	$\sum_{n=1}^{\infty}$	$\frac{1}{n}$ sa	tisfy the	follow	ing inequ	ality											

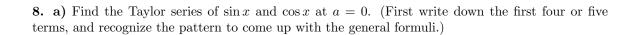
$$s_n \le 1 + \ln n.$$

b) Even though the harmonic series diverges, it does so $very\ slowly$. Use part a) to show that the sum of the first million (10^6) terms is less than 15. (you may use the fact that $\ln 10 = 2.32...$)



b) Find an approximate value of $e^{0.9}$ by using the second order Taylor's polynomial T_2 of e^x centered at a=1.

c) Find an upper bound for the error $R_2(0.9) = e^{0.9} - T_2(0.9)$ for the approximation of the part a. Hint: You can either use Alternating Series Error Estimate or Taylor's Theorem.



b) Differentiating the power series for $\sin x$ from a) and check that it equals to the power series for $\cos x$ as you expect (recall $(\sin x)' = \cos x$.)

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