MA 125-CV,   CALCULUS I
January 30, 2007

Name (Print last name first): . . . .

 Student ID Number (last four digits): . . . .

$$\boxed{\text{TEST I}}$$

$$\boxed{\text{PART I}}$$

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. No partial credit is awarded for this part of the test!

Each question is worth 5 points.

Question 1

The function
$$f(x) = \frac{x^2 - 2x - 8}{x + 2}$$
has a discontinuity at the number $x = -2$. Is this number a removable discontinuity or is it an infinite discontinuity?

Answer: . . . .

Question 2

Given that $\lim\limits_{x \to a} f(x) = -1$ and $\lim\limits_{x \to a} h(x) = 3$, find
$$\lim_{x \to a} \frac{2f(x)}{f(x) - h(x)}.$$

Answer: . . . .

Question 3

Use the definition of continuity to evaluate [note that your answer must be a number]

$$\lim_{x \to 0} \cos\left(x + \sin x\right).$$

Answer: ....

Question 4

If $3x - 2 \le g(x) \le 2x^2 - x$ for all x, evaluate

$$\lim_{x \to 1} g(x)$$

if the limit exists.

Answer: ....

Question 5

Find the limit

$$\lim_{x \to \infty} \frac{7x - x^4}{3x^4 - x^3 + 1}.$$

Answer: ....

Question 6

Given that $f(2) = -1$ and $f'(2) = -2$, find an equation of the tangent line to the graph of $y = f(x)$ at $x = 2$.

Answer: ....

PART II

Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

Consider the function
$$f(x) = \begin{cases} 3x + 5 & \text{for } x < -2, \\ 3 - x^2 & \text{for } x \geq -2. \end{cases}$$

(a) Evaluate
$$\lim_{x \to -2^-} f(x).$$

(b) Evaluate
$$\lim_{x \to -2^+} f(x).$$

(c) Is this function continuous at $x = -2$? (Justify your answer!)

Problem 2

Sketch the graph of an example of a function f such that

$$\lim_{x \to 0^-} f(x) = 0, \qquad \lim_{x \to 0^+} f(x) = 1, \qquad f(0) = 1,$$

$$\lim_{x \to 2^-} f(x) = 1, \qquad \lim_{x \to 2^+} f(x) = -1, \qquad f(2) = 0.$$

Problem 3

(a) Evaluate the limit

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 3x + 2}$$

(b) Evaluate the limit

$$\lim_{x \to 9} \frac{\sqrt{x + 7}}{-1 + \sqrt{x}}$$

Problem 4

(a) For what (numerical) value of the constant c is the function

$$f(x) = \begin{cases} x^2 - cx & \text{if } x > -1 \\ cx^3 + x & \text{if } x \leq -1 \end{cases}$$

continuous on $(-\infty, \infty)$? (Justify your answer!)

(b) The displacement (in feet) of a particle moving in a straight line is given by the equation of motion $s = \sqrt{t} + 3$, where t is measured in seconds. Find the velocity of the particle at time $t = 9$ seconds.

Problem 5

Let $G(x) = \dfrac{2}{x+1}$.

(a) Use **the limit definition of the derivative** to find $G'(0)$.

(b) Find an equation of the tangent line to the curve $y = \dfrac{2}{x+1}$ at the point $(0, 2)$.