MA 125 - 6C, CALCULUS I
November 4, 2009

Name (Print): ....

Student Signature: ....

TEST III

No calculators are allowed!

PART I

Part I consists of 10 questions. Clearly write your answer in the space provided after each question. Show your work as much as possible and simplify your answer when possible

Each question is worth 5 points.

Question 1

Differentiate the function $y = \ln(3x - 5x^2)$

Answer: $y' = \frac{3-10x}{3x-5x^2}$

Question 2

Differentiate the function $y = e^{-x^3}$

Answer: $y' = -3x^2 e^{-x^3}$

Question 3

Differentiate the function $y = \tan^{-1}(5x+1)$

Answer: $y' = \frac{5}{1+(5x+1)^2}$

Question 4

Differentiate the function $y = x^2 \sin^{-1}(x+1)$

Answer: $y' = 2x\sin^{-1}(x+1) + \frac{x^2}{\sqrt{1-(x+1)^2}}$

Question 5

Differentiate the function $y = \sqrt{e^x - 4}$

Answer: $y' = \frac{e^x}{2\sqrt{e^x-4}}$

Question 6

Evaluate $\lim\limits_{x\to\infty} \dfrac{100+\ln x}{x}$

Answer: 0

Question 7

Find the linearization of the function $f(x) = \sqrt{1 + 2x}$ at $a = 0$

Answer: $L(x) = 1 + x$

Question 8

Simplify $\tan\bigl(\cos^{-1}(x)\bigr)$

Answer: $\frac{\sqrt{1-x^2}}{x}$

Question 9

Differentiate the function $y = \ln\bigl(\tan(x)\bigr)$

Answer: $y' = \frac{1}{\sin x \cos x}$

Question 10

Use Newton's method two find the second approximate solution x_2 to the equation

$$x^3 - x - 4 = 0$$

if $x_1 = 1$.

Answer: $x_2 = 3$

PART II

Each problem is worth 10 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit. Simplify when possible

Problem 1

Use logarithmic differentiation to find the derivative of

$$y = \frac{x^{7/4}(x^2+x)^{12}}{\sqrt[3]{6x-1}}$$

You do not need to simplify the answer, but you must express your answer in x (and not in y).

Answer:

$$y' = \frac{x^{7/4}(x^2+x)^{12}}{\sqrt[3]{6x-1}}\left(\frac{7}{4x} + \frac{12(2x+1)}{x^2+x} - \frac{2}{6x-1}\right)$$

Problem 2

Find the derivative y' if

$$y = x^{\ln(x^2)}$$

Answer:

$$y' = x^{\ln(x^2)}\left(\frac{4\ln x}{x}\right)$$

Problem 3

(a) Find the limit

$$\lim_{x\to 1} \frac{\ln x}{1-\sqrt{x}}$$

Answer: -2

(b) Find the limit

$$\lim_{x\to 0} (1-\sin x)^{2/x}$$

Answer: e^{-2}

Problem 4

Find the limit

$$\lim_{x \to 0} \frac{1 - \cos(4x)}{x^2}$$

Answer: 8.

Problem 5

An ideal cubic box with side 5m has volume 125m^3 (because $5^3 = 125$). Suppose the side of a cubic box is measured as 5m with an error of 0.01m.

(a) Use differentials to approximate the error in the volume.

Answer: 0.75

(b) Use (a) to find the relative error of the volume.

Answer: 0.006

(c) Use (b) to find the percentage error of the volume.

Answer: 0.6%.