Name:____________________ Signature:____________________

Exam IV

Calculus I; Fall 2009

Part I

Part I consists of 10 questions, each worth 5 points. Clearly show your work for each of the problems listed.

(1) Let $f(x) = 4x^3 - 12x + 2$. Find all local max/min of $f(x)$. State both x and y coordinates.
Answer: local max at $(-1, 10)$ *and local min at* $(1, -6)$.

(2) Find the absolute max/min of $f(x) = 10 - x^2$ on the interval $[-1, 2]$. Give both x and y-coordinates and justify your answer.
Answer: absolute max at $(0, 10)$ *and absolute min at* $(2, -6)$.

actually, (2,6)

(3) Find two positive numbers whose product is 100 and whose sum is minimal. (You must justify your answer.)
Answer: 10 and 10.

(4) Let $f'(x) = (x-2)^2(x-1)(x+1)$. **Note that you are already given the derivative $\mathbf{f'(x)}$.** Find all critical points, where $f(x)$ is increasing and decreasing, and also find the x-coordinate(s) of all local max/min.
Answer: increasing on $(-\infty, -1)$ and $(1, \infty)$, decreasing on $(-1, 1)$. Local min at $x = 1$, local max at $x = -1$.

(5) If $f''(x) = (x-1)^2(x+3)$ find where $f(x)$ is concave up and where it is concave down. Also find all points of inflection. **Note that you are given $\mathbf{f''(x)}$!**
Answer: concave up on $(-3, \infty)$, concave down on $(-\infty, -3)$, inflection point at $x = -3$.

(6) Find the most general **anti-**derivative of $\frac{3-x+4\sqrt{x}}{x^3}$.
Answer: $-\frac{3}{2}x^{-2} + x^{-1} - \frac{8}{3}x^{-3/2} + C$.

(7) Find the most general **anti-**derivative of $\cos(x) - \frac{1}{x}$.
Answer: $\sin x - \ln|x| + C$.

(8) Find all asymptotes of the function $\frac{x^3+5}{x(x-2)(x+1)}$.
Answer: vertical: $x = 0,\ x = 2,\ x = -1$ *and horizontal* $y = 1$.

(9) If the acceleration is given by $a(t) = 6t$, $v(0) = 2$ and $s(0) = 1$, find an expression for the position $s(t)$.
Answer: $s(t) = t^3 + 2t + 1$.

(10) If $f(x) = x^3$ find the number c that satisfies the conclusion of the mean value theorem on the interval $[0, 2]$.
Answer: $c = 2/\sqrt{3}$.

Part II

Part II consists of 3 problems; the number of points for each part are indicated by [x pts]. You must show the relevant steps (as we did in class) and justify your answer to earn credit. Simplify your answer when possible.

(1) [**15 pts**] Find the absolute max/min of the function $f(x) = (x^2-1)^3$ on the interval $[-2, 3]$.
Answer: absolute min $(0, -1)$, *absolute max* $(3, 512)$.

(2) Given the function $f(x) = \frac{x^2-9}{x^2-1}$,

(a) [**2 pts**] Find the x and y intercepts of the function.
Answer: $x = \pm 3$ *and* $y = 9$.

(b) [**3 pts**] Find all asymptotes.
Answer: vertical $x = 1$ *and* $x = -1$, *horizontal* $y = 1$.

(c) **[4 pts]** Find the open intervals where $f(x)$ is increasing and the open intervals where $f(x)$ is decreasing,
Answer: increasing on $(0, 1)$ *and* $(1, \infty)$, *decreasing on* $(-\infty, -1)$ *and* $(-1, 0)$.

(d) **[2 pts]** Find the local maximum and local minimum value(s) of $f(x)$. (Be sure to give the x and y coordinate of each of them).
Answer: local min at $(0, 9)$, *no local max.*

(e) **[2 pts]** Find all open intervals where the graph of $f(x)$ is concave up and all open intervals where the graph is concave down.
Answer: concave up on $(-1, 1)$, *concave down on* $(-\infty, -1)$ *and* $(1, \infty)$.

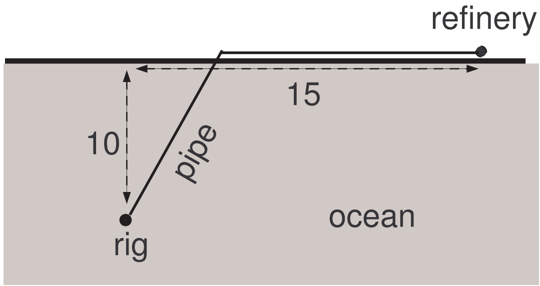
(f) **[2 pts]** Find all points of inflection (be sure to give the x and y coordinate of each point).
~~*Answer:* $x = 1$ *and* $x = -1$.~~

Correct answer: no inflection points.

(g) **[6 pts]** Use the above information to graph the function **on the next page**. Indicate all relevant information in the graph.

Put the graph of Problem 2 on this page.

(3) [**14 pts**] A drilling rig in the ocean is 10 mi of shore. A refinery is located along the coast 15 mi away from the point on the shore closest to the rig. If under water pipe lines cost \$5 per mi and land-based pipe costs \$4 per mi, what is the least expensive way to run the line.



Partial answer: The total cost of the pipe is

$$f(x) = 4(15 - x) + 5\sqrt{10^2 + x^2}$$

you need to find x where this function is a minimum.