

EGR 265, Math Tools for Engineering Problem Solving
October 14, 2009, 50 minutes

Name:

TEST II

Problem 1 (20 points)

Solve the initial value problem

$$y'' + 6y' + 9y = 0, \quad y(0) = 2, \quad y'(0) = -2.$$

Problem 2 (20 points)

Find the general solution of

$$y'' - 2y' + 5y = 2x.$$

Problem 3 (20 points)

Find the general solution of

$$y'' - y' - 2y = -10 \sin(2x).$$

Problem 4 (20 points)

A mass of 3 kilograms stretches an undamped spring by 98 centimeters.

- (a) Find the value of the spring constant k . Include the correct unit of k (using the mks-system).
- (b) Find the frequency $\frac{\omega}{2\pi}$ of free oscillations of the spring/mass-system.
- (c) Find the equation of motion if the mass is released from rest at a position 50 centimeters below the equilibrium. Assume here that the positive x -direction is oriented downwards.
- (d) Find the first positive time at which the mass passes through the equilibrium position. (Note: In the example considered here, there is a simpler way to answer this than the method used in class.)

Problem 5 (10 points)

Suppose that a damping force is added to the spring/mass system in Problem 4 which is proportional to the instantaneous velocity with damping coefficient $\beta = 15$ kg/sec. Does the resulting system become underdamped, critically damped, or overdamped? Justify your answer.

Problem 6 (10 points)

Find the largest interval containing $x = 1$ for which the IVP

$$y'' - \frac{1}{x^2 - 4x}y = \sqrt{3 - x}, \quad y(1) = -1, \quad y'(1) = 3$$

has a unique solution.