Final Exam **Calculus 1** **Fall 2013**

Name: _____ **Signature:** _____

SHOW ALL YOUR WORK!

If you have time, find a way to check your answers.

Part 1

1. [5 points] Evaluate $\lim_{t\to\infty} \dfrac{5t^3+7t^3+6}{2t^3-7t-5}$

2. [5 points] Given that $\lim_{t\to a} h(t) = -6$ and $\lim_{t\to a} g(t) = -9$, find

$$\lim_{t\to a} \frac{h(t)}{g(t)-h(t)}$$

3. [5 points] Find the x-coordinate of each critical number of $f(\theta) = 2\cos(\theta) - \sin^2(\theta)$ in $[0, 2\pi]$.

4. [5 points] Find the values of x for which the curve $y = 2x^3 - 9x^2 - 24x + 1$ has a horizontal tangent line.

5. [5 points] Find the linearization $L(x)$ of the function $f(x)$ at $\frac{\pi}{4}$ for $f(x) = \sin(x)$

6. [5 points] Find the second derivative of the function $f(x) = \cos(x^3)$.

7. [5 points] Find y' if $\cos(xy) = 1 + \sin(y)$.

8. [5 points] Differentiate $f(x) = e^{\sin x \cos x}$.

9. [5 points] Find $\lim_{x \to 0} \dfrac{e^x - 1 - x}{x^2}$.

10. [5 points] A particle moves along a straight line so that its coordinate at the time t is $s(t)$. It is known that its velocity equals $v(t) = 6t^2 + 4t + 1$ and that $s(0) = 0$. Find the precise expression for $s(t)$.

Part 2

1. [5 points] Differentiate $f(x) = e^{\tan x} - (\ln(x))^5$

2. [5 points] Given the following function on the given interval

$$g(t) = t^2 + 2t + 1, \qquad [-2, 0],$$

find all numbers c that satisfy the conclusion of the Mean Value Theorem.

3. [5 points] Use logarithmic differentiation to calculate the derivative of

$$y = \frac{x^{\frac{3}{4}}\sqrt{x^2+4}}{(3x+4)^5}$$

4. [10 points] If $y = f(x) = -4x\sqrt{x+3}$, find the absolute maxima and minima of $f(x)$ on the closed interval $[-3, 6]$. Include the appropriate x and y values of the maximum and minimum.

5. [8 points] Find the dimensions of a rectangle whose area is 9 and whose perimeter is minimal.

6. Let $f(x) = \dfrac{x-2}{(x-1)^2}$.

   (a) [2 points] Find the domain and the x and y intercepts of the function.

(b) [3 points] Find the vertical and horizontal asymptotes of the function.

(c) [2 points] Find the open intervals where $f(x)$ is increasing and the open intervals where $f(x)$ is decreasing.

(d) [2 points] Find the local maxima and the local minima of the function if any (give both x and y coordinates of each of them).

(e) [2 points] Find all open intervals where the graph of $f(x)$ is concave up and all open intervals where it is concave down.

(f) [1 points] Find all inflection points (give both x and y coordinates!).

(g) [5 points] Use all this information to graph the function. Indicate all relevant information on the graph (such as x, y-intercepts, local/absolute maxima/minima, asymptotes, inflection points etc).