MA 126 - 8C CALCULUS II

December 01, 2016

Name (Print last name first): . . . .

Student Signature: . . . .

TEST IV

Closed book - Calculators and One Index Card are allowed!

PART I

Part I consists of 5 questions. Clearly write your answer (only) in the space provided after each question. Show your work to justify your answers. Very limited partial credit or none at all for this part of the test!

Each question is worth 8 points.

Question 1

Find both the scalar projection $\text{comp}_{\mathbf{v}}\mathbf{u}$ and the vector projection $\text{proj}_{\mathbf{v}}\mathbf{u}$ of the vector $\mathbf{u} = \langle 1, 1, 1\rangle$ onto the vector $\mathbf{v} = \langle 1, 0, 1\rangle$.

Answer: . . . .

Question 2

A constant force with vector representation $\mathbf{F} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ moves an object along a straight line from the point $P(1,1,1)$ to the point $Q(1,2,1)$. Find the work done if the distance is measured in meters and the magnitude of the force is measured in newtons.

Answer: ....

Question 3

Find the parametric equations of the line that passes through the point $P(-1,1,0)$ and that is perpendicular to the vectors $\mathbf{u} = \langle 1,0,1 \rangle$ and $\mathbf{v} = \langle 0,1,1 \rangle$.

Answer: ....

Question 4

Find an equation of the plane containing the point $P(1, -1, 1)$ and which is parallel to the plane $x + 2y - z = 1$.

Answer: ....

Question 5

Find, if any, (the coordinates of) the point of intersection of the plane $x - y - z = 2$ and the line given by the parametric equations

$$\ell := \begin{cases} x = 3 - t \\ y = 1 - t \\ z = -1 + t \end{cases}$$

Answer: ....

PART II

Each problem is worth 15 points.

Part II consists of 4 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit - no credit for unsubstantiated answers!

Problem 1

This problem has two separate questions (a) and (b). Answer each question.

(a) Find the area of the triangle with vertices $P(2, -1, 3)$, $Q(3, -1, 4)$ and $Q(2, 0, 4)$.

(b) Find the volume of the box generated by the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + \mathbf{k}$ and $\mathbf{c} = \mathbf{j} + \mathbf{k}$.

Problem 2

Consider the two lines given by the parametric equations

$$\ell_1 = \begin{cases} x = 2 + t \\ y = -1 - t \\ z = 1 + 2t \end{cases} \quad \text{and} \quad \ell_2 = \begin{cases} x = 1 - 2s \\ y = -1 + s \\ z = 2 - s \end{cases}$$

(a) Determine whether they are parallel.

(b) Determine whether they intersect. If they do intersect, find the point of intersection.

(c) Determine whether they are skew.

Problem 3

Determine whether or not the planes given below intersect. If they do intersect, find both the parametric equations and the symmetric equations of the line of intersection.

$$x + y - z = 1 \quad \text{and} \quad x - y - z = 1.$$

Problem 4

A particle is traveling along the space-curve

$$\mathbf{r}(t) = \langle\, 3t,\ -4\,\cos(t),\ 4\,\sin(t)\,\rangle$$

when the time t is such that $-\infty < t < \infty$.

(i) Determine the velocity-vector of the particle at the time $t = 0$.

(ii) Find the unit tangent vector to this space-curve at the point where $t = 0$.

(iii) Find the arc-length of this curve (i.e., the distance traveled by the particle) for the period $1 \le t \le 5$.

SCRATCH PAPER

(Scratch paper will not be graded!)

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