MA 126-6A Spring 2003 Final Examination
Name _____

1. Consider

$$\int_2^4 (2x - 3x^2)dx.$$

(a) Write a Riemann sum for this integral.
(b) Evaluate this integral by taking a limit of Riemann sums.
Reference: The following formulas may be useful:

$$\sum_{j=1}^{n} j = \frac{1}{2}n(n+1), \sum_{j=1}^{n} j^2 = \frac{1}{6}n(n+1)(2n+1),$$

$$\sum_{j=1}^{n} j^3 = \frac{1}{4}n^2(n+1)^2$$

2. Evaluate each of the following integrals.

(a)

$$\int e^{-2x}\sin(3x)dx$$

(b)

$$\int_{-\pi/4}^{\pi/4} \cos(t)\ln(\sin(t))dt$$

3. Let R be the region bounded by the graphs of $y = x^2$ and $x = y^2$.
(a) Find the volume of the solid formed by revolving R about the $x-$ axis.
(b) Find the volume of the solid formed by revolving R about the $y-$ axis.
4. For each of the following series, give a reason why the series converges, or a reason why it diverges.
(a)

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^{1/4}}$$

(b)

$$\sum_{n=0}^{\infty}\frac{n^4}{5^n}$$

(c)

$$\sum_{n=2}^{\infty}\frac{1}{n\ln(n)}$$

5. Let

$$f(x) = \frac{3}{6 + 9x}.$$

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(a) Find the power series expansion of $f(x)$ about -5.

(b) Find the radius of convergence of the power series expansion of $f(x)$ about -5.

(c) Find the interval of convergence of the power series expansion of $f(x)$ about -5.

6. Let
$$f(x) = x^2 e^{-x}.$$

(a) Find the power series expansion of $f(x)$ about 0.

(b) Determine $T_4(x)$, the fourth Taylor polynomial of $f(x)$ about 0.

(c) Determine a bound on the error if $f(x)$ is approximated by $T_4(x)$ for $-1/3 \le x \le 1/3$.