TEST # 3

March 31, 2004 (70 minutes)

Name: SSN:

Max. Points: 100 + 5 Bonus   Points: Test Grade:

Turn in **all the work** which you did to solve the problems, not just the final answer. In particular, include **intermediate steps in calculations**, wherever they are needed. You may write on the back of a page if you need extra space.

No book, no notes, and no calculator are to be used!

1. Find $f'(x)$ for the following functions $(3 \times 5\text{P})$:

(a) $f(x) = xe^{3x}$

(b) $f(x) = \ln(x + \sqrt{x})$

(c) $f(x) = \left(\dfrac{\sin x}{\cos x - 1} \right)^2$

2. Evaluate the following limits $(3 \times 5\text{P})$:

(a) $\displaystyle\lim_{x \to 0} \dfrac{e^x - 1 - x}{x^2}$

(b) $\displaystyle\lim_{x \to \infty} e^{-x} \ln x$

(c) $\displaystyle\lim_{x \to 1} \dfrac{x^2}{x + 1}$

3. (a) Find the linearization of $f(x) = \sqrt{4+x}$ at $a = 0$. (8P)

(b) Use the result from (a) to find an approximate value for $\sqrt{4.2}$. (4P)

4. (a) In the graphs below use Newton's Method to graphically determine an approximation x_2 for the root of $f(x)$ from the given rough approximation x_1. Also note if the given values for x_1 are good choices to start Newton's Method. (8P)

(b) Use Newton's Method to find an approximation x_2 for a root of $f(x) = x^2 - 2$, given the rough approximation $x_1 = 2$. (5P)

5. Find the absolute minimum and maximum values of $f(x) = x - 2\sqrt{x}$ on the interval $0 \leq x \leq 9$. (12P)

6. (a) State the Mean Value Theorem. (3P)

(b)* Check if the following is possible: A differentiable function $f(x)$ on $[0, 1]$ has the property that $f(0) = 1$, $f(1) = 3$ and $f'(x) \leq 1$ for all x in $[0, 1]$. If this is possible, find such a function. If it is not possible, explain why such a function does not exist. (5P*)

7. Let $f(x) = x^3 - 3x^2$. (total: 20P)

   (a) Find the intervals in which f is increasing/decreasing. (4P)

   (b) Find all local extreme values of f. (3P)

   (c) Determine the intervals where f is concave up, concave down. (4P)

   (d) Find the points of inflection of f. (3P)

   (e) Sketch the graph of f. (6P)

8. A farmer wants to fence off a rectangular field with an area of 800 m². One side of the field borders to a river where no fence is needed. She wants to buy as little fence as possible. How much fence is needed? (10P)