### Test 2

# Calculus III - MA 227 7B February 28, 2007

## Time: 60 min

#### Instructions

This test consists of two parts.

In Part I, your answer must be correct. No partial credit will be given. For credit to be awarded in this part you **MUST** show your work. Write your answer in the space indicated. There are 5 questions in this part, each worth 3 points, for a total of 15 points.

In Part II, give a <u>complete</u> answer to the question in the space provided. In this part partial credit may be awarded. There are 4 questions in this part. Question 6 is worth 5 points and Questions 7 through 9 are each worth 10 points, for a total of 35 points.

You may use the back of the pages if the space next to questions is insufficient. The back pages may also be used for scratch work. However clearly indicate so if you are using it for scratch. Leave all test sheets stapled.



## Part I

Write your answers in the place indicated

**Question 1.** Find the domain of the function  $f(x, y) = \ln(16 - x^2 - 16y^2)$ . Enter you answer in the set notation i.e.  $D = \{(x, y) | \dots \}$ .

Answer:

Question 2. Find the first partial derivatives of the function  $z = \frac{x^2}{y} - \frac{y^2}{x}$ .

Answer:

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Question 3. Find the directional derivative of  $f(x, y, z) = x^2 + y^2 + z^2$  at P(2, 1, 2) in the direction of the origin.

Answer: \_\_\_\_\_\_ Question 4. Find the maximum rate of change of  $f(x, y, z) = \ln(xy^2z^3)$  at

(1, -2, -3).

Answer:

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**Question 5.** Find the derivative of f(x, y) in the direction  $u = \left\langle \frac{\alpha}{\sqrt{\alpha^2 + \beta^2}}, \frac{\beta}{\sqrt{\alpha^2 + \beta^2}} \right\rangle$  at the *critical point* (a, b).

Answer:

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# Part II

Answer in the space provided

Question 6. Find the linear approximation of the function

$$f(x,y) = \ln(x^2 - 3y^2)$$

at (2,1) and use it to approximate f(1.9,1.1).

(5 points)

Question 7. Use Chain Rule to find  $\frac{\partial z}{\partial u}$ ,  $\frac{\partial z}{\partial v}$  and  $\frac{\partial z}{\partial w}$  where,  $z = x^2 + xy^3$   $x = uv^2 + w^3$   $y = u + ve^w$ when u = 2, v = 1, w = 0 Question 8. Find and classify the critical points of the function  $f(x,y) = x^3 + xy^2 + 6x^2 + y^2.$ 

**Question 9.** Use Lagrange multipliers to find the maximum and minimum values of the function

$$f(x, y, z) = 16x - 8z$$

subject to the constraint

$$x^2 + 15y^2 + z^2 = 20.$$