EGR/MA 265, Math Tools for Engineering Problem Solving
April 14, 2010, 50 minutes

Name (Print Last Name First): .....

Student Signature: .....

TEST III

Problem 1

(a) (9 pts) Let $f(t,x) = \dfrac{t}{t^2 - x^2}$. Find $f_{tt} - f_{xx}$.

(b) (9 pts) For the function $g(x,y) = y\sin(x^2 y)$ find g_x, g_y and g_{xy}.

Problem 2

(a) (9 pts) For the function $h(x, y) = xe^y$ find its direction **and** rate of steepest ascent at the point $P(2, 0)$.

(b) (9 pts) Find the directional derivative of $h(x, y)$ at $P(2, 0)$ in the direction of the vector from $P(2, 0)$ to $Q(\frac{1}{2}, 2)$.

Problem 3

(a) (12 pts) Find an equation for the tangent plane to the level surface

$$x^2 - 2y^2 + z^2 + yz = 2$$

at the point $(2, 1, -1)$.

(b) (6 pts) Also, find parametric equations for the normal line to the level surface

$$x^2 - 2y^2 + z^2 + yz = 2$$

at the point $(2, 1, -1)$.

Problem 4 (12 pts)

Evaluate $\int_C xy^4\,ds$ where C is the right half of the unit circle $x^2 + y^2 = 1$.

Problem 5 (12 pts)

Find the work done by the force field

$$F(x, y) = x^2 y\,\mathbf{i} - y\sqrt{x}\,\mathbf{j}$$

along the curve C parameterized by $x = t^2$, $y = t$, $0 \le t \le 1$.

Problem 6

Determine for each of the following force fields if it is conservative.

(a) (5 pts) $F(x, y) = (x^3 + 4xy)\mathbf{i} + (4xy - y^3)\mathbf{j}$

(b) (5 pts) $F(x, y) = x^3y^4\,\mathbf{i} + x^4y^3\,\mathbf{j}$

Problem 7 (12 pts)

For the conservative force field $F(x, y)$ from Problem 6 find a potential function $\phi(x, y)$ and calculate the work done by the force field along the curve traced by the vector function $\mathbf{r}(t) = \sqrt{t}\,\mathbf{i} + (1 + t^3)\mathbf{j}$, $0 \le t \le 1$.

SCRATCH PAPER

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