

MA-227/6D: CALCULUS III  
TEST #1, FEBRUARY 13, 2012

Time available: 110 min

Your name (print):

Your signature:

**Please always explain your answer, at least by including your calculations. You should work on this sheet. A right answer without calculation brings you no credit!**

1. At what point(s) does the curve

$$\vec{r}(t) = \langle t - 2, 11t^2 - 11t + 14, 3t \rangle$$

intersect the paraboloid  $y = 3x^2 + z^2$ ?

10 points

2. Find a parametric equation for the tangent line to the previous curve at the point  $P(0, 36, 6)$ .

10 points

2

3. Find the curvature of the curve  $\vec{r}(t) = \cos t\vec{i} + \cos t\vec{j} - 3\sin t\vec{k}$  at the point  $P(1, 1, 0)$ .

20 points

4. Find the vectors  $T$ ,  $N$ , and  $B$  for the curve of problem 3 at the given point.

20 points

5. Find the tangential and normal components of the acceleration vector for the curve  $\vec{r}(t) = t\vec{i} + 2t\vec{j} + t^2\vec{k}$  at the generic point  $\vec{r}(t)$ .

10 points

6. The motion  $\vec{r}(t)$  takes place for positive time (always  $t > 0$ ),  $\vec{a}(t) = 6t\vec{i} + \frac{1}{t^2}\vec{j} + 6t\vec{k}$ ,  $\vec{v}(1) = 3\vec{i} - \vec{j} + 3\vec{k}$ ,  $\vec{r}(1) = \vec{i} + \vec{k}$ . Compute  $\vec{r}(t)$ .

20 points

4

7. Find the vectors  $\vec{T}(t)$ ,  $\vec{N}(t)$ , and  $\vec{B}(t)$  for the curve

$$\vec{r}(t) = \langle 3t, 4 \cos t, 4 \sin t \rangle.$$

10 points

8. Find the velocity, acceleration, and speed of a particle with the position function

$$\vec{r}(t) = \langle t, t \sin t, -t \cos t \rangle.$$

10 points

9. Find the curvature of the space curve

$$\vec{r}(t) = t\vec{i} + t\vec{j} - t^2\vec{k}$$

at any point.

10 points

10. Let  $r(t) = \langle t, 3t, t^2 \rangle$ . Find the tangential and normal components of the acceleration, i. e. find  $a_T$  and  $a_N$ .

10 points